Damiers de phase diophantiens Pour l'Exoplanétologie (future)

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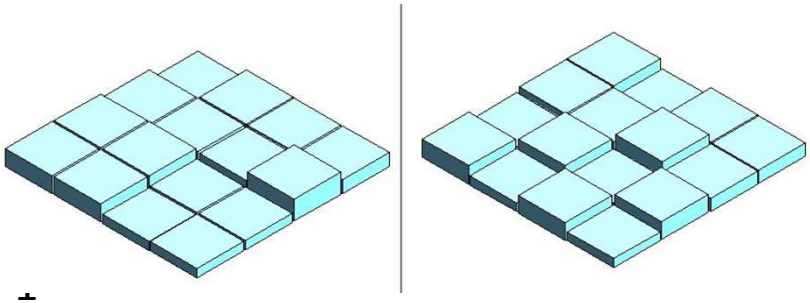
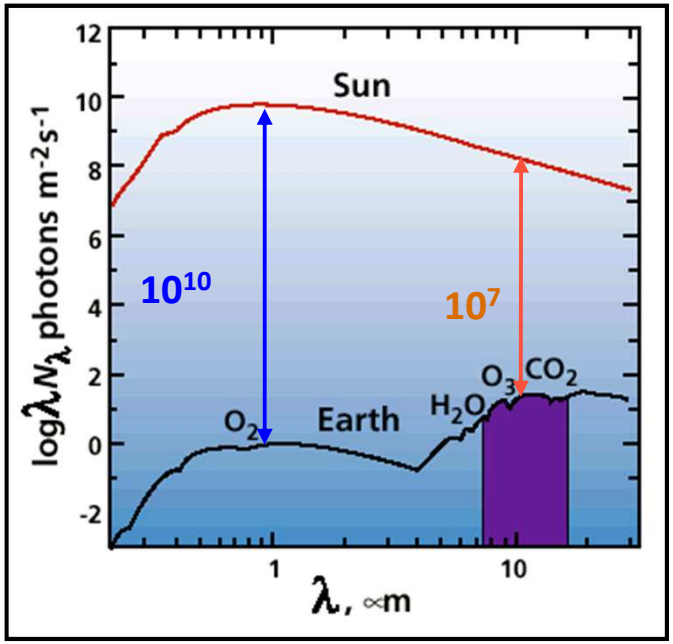
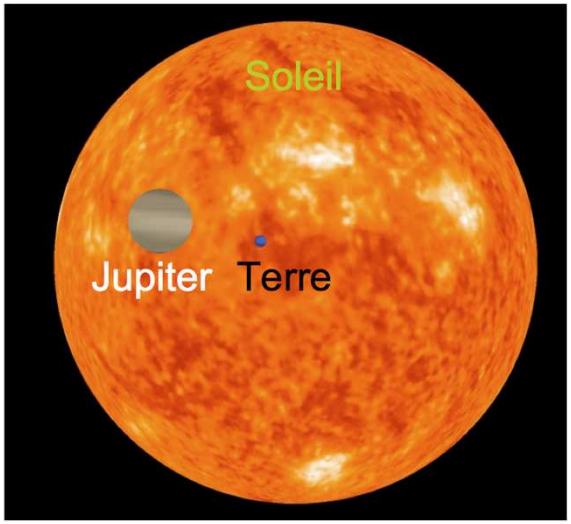
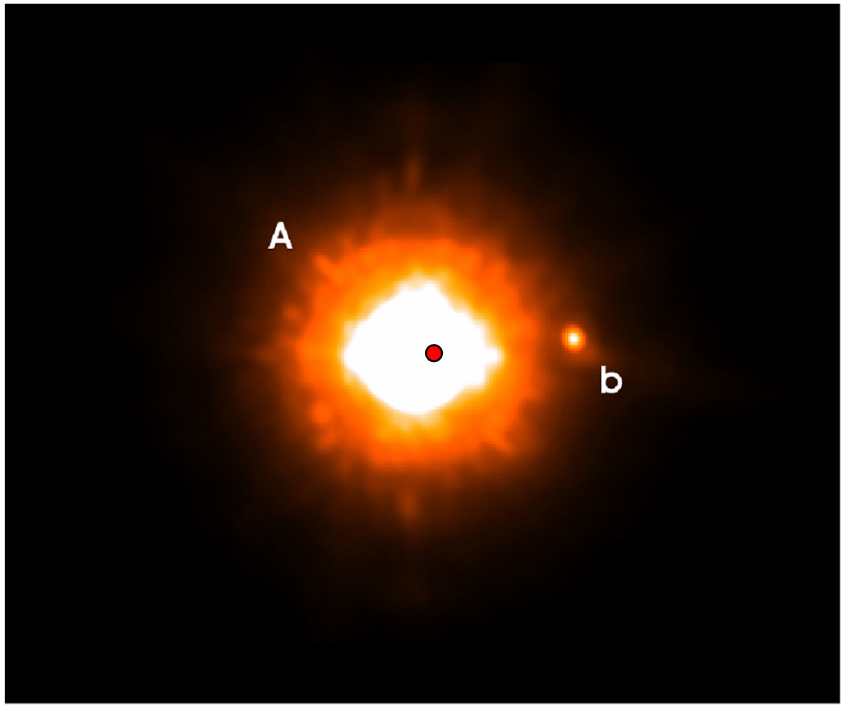


Detecting an exoplanet ?







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Detecting an exoplanet ?

Detecting *directly* an exoplanet is TERRIBLY difficult !!!

Exoplanet direct detection

- Direct detection and characterization of a planet is thus a hard task
- Two avenues :
  - In visible : one telescope + adaptive optics + coronagraphy
  - In infrared : multi-telescopes + space + nulling interferometry

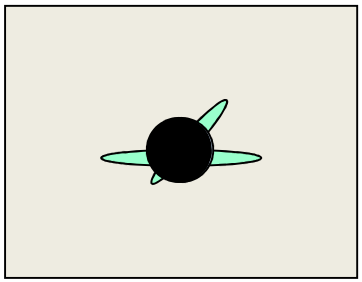
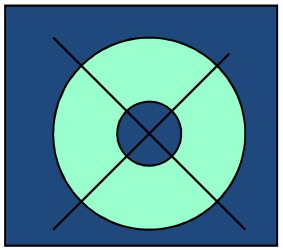
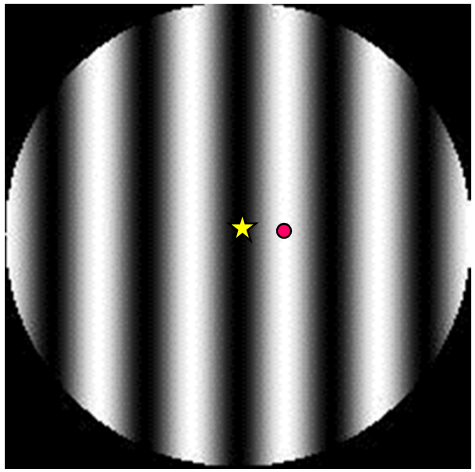


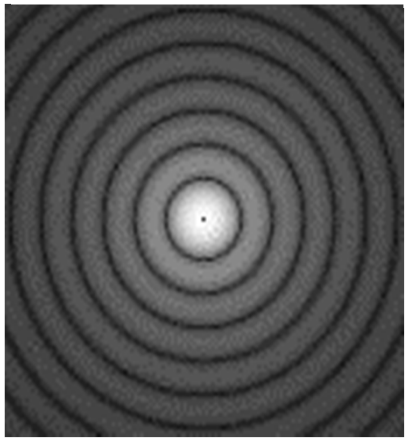
Image plane



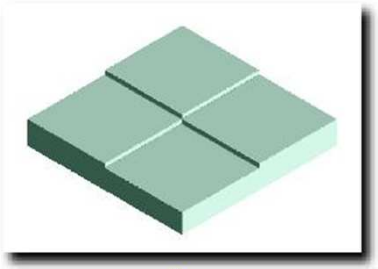
Pupil plane

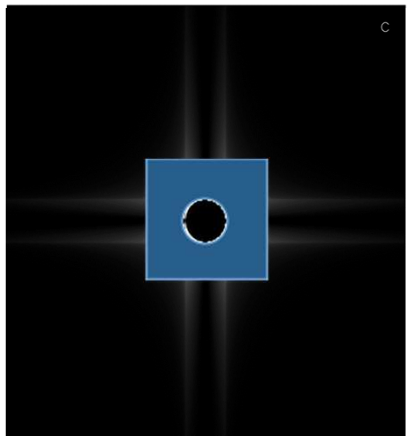


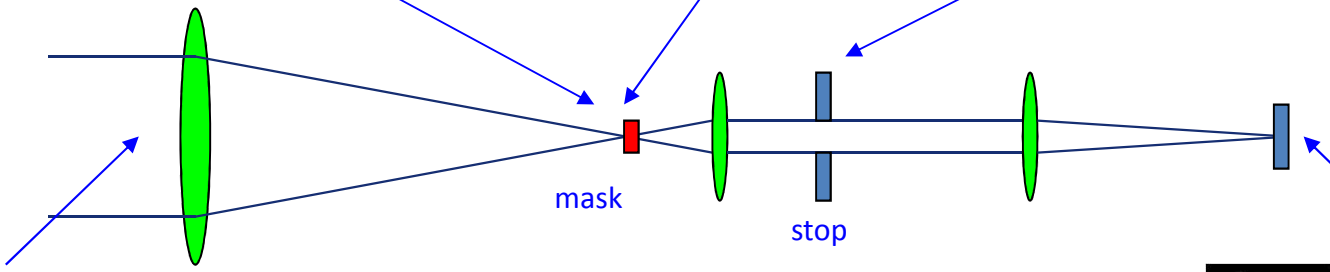
4 quadrants phase mask coronagraph

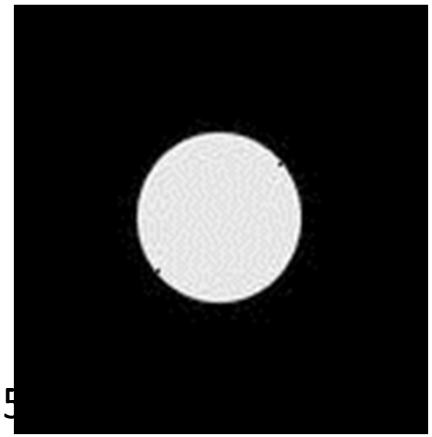


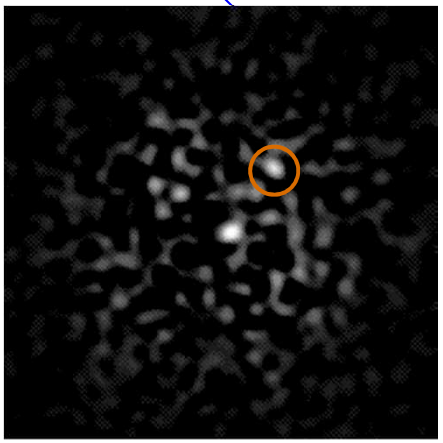
A transparent mask











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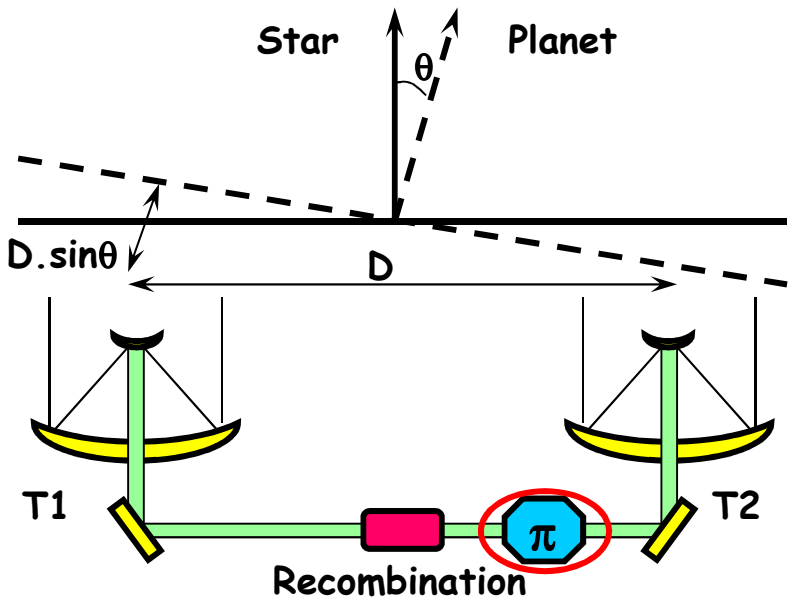
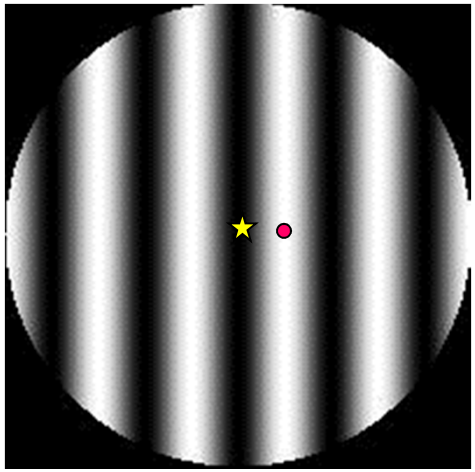
Nulling interferometry

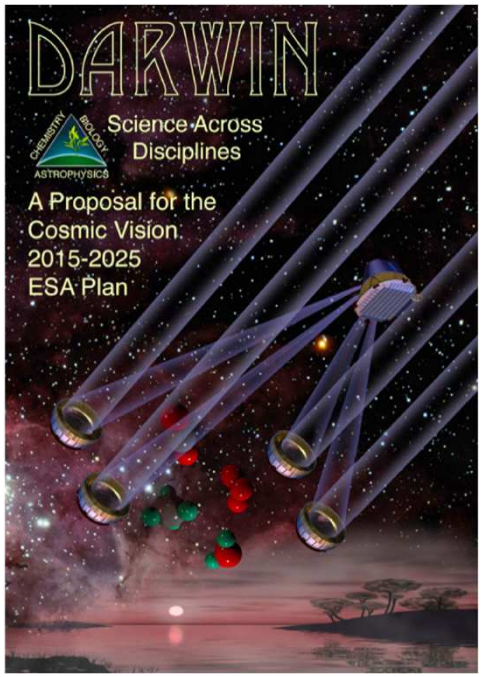
- Bracewell-type nulling interferometer
- Success depends on several challenges, one being the *achromatic* π *phase shift* within the bandpass (typically one to two octave)



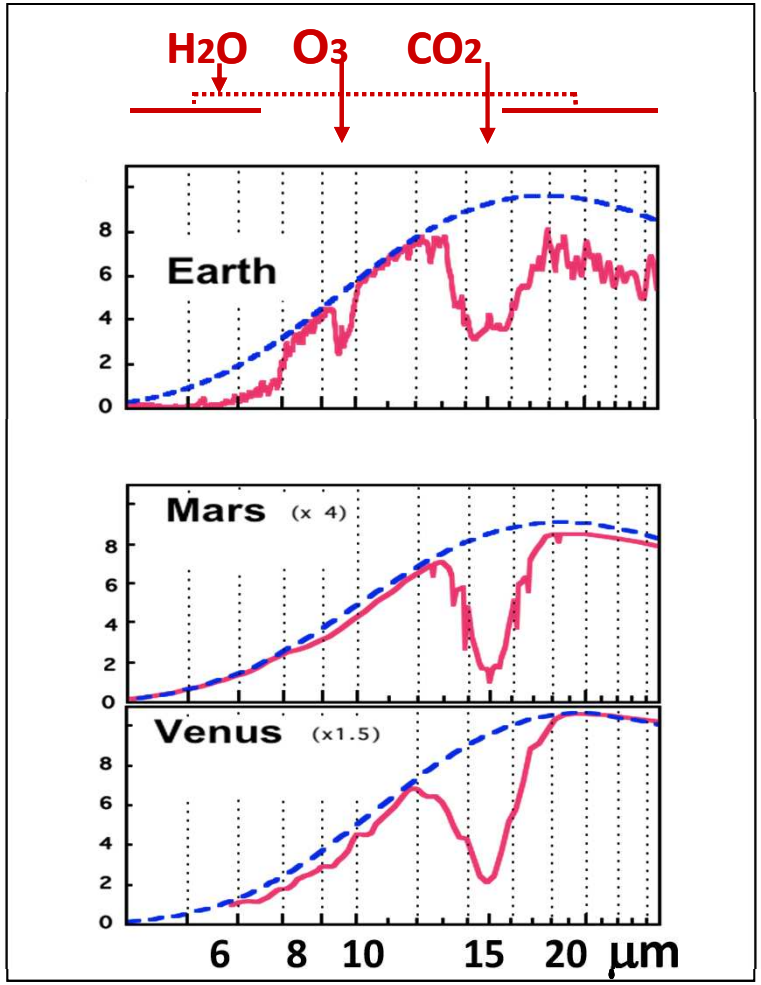


Life on exoplanet: tracers in the mid-IR

- Signatures of three key components: H_2O, O_3, CO_2
- Broad range of ⍰ : from 6 to 18 µm
- Detecting those bands : goal of a future Darwin-type space project



Observed spectra

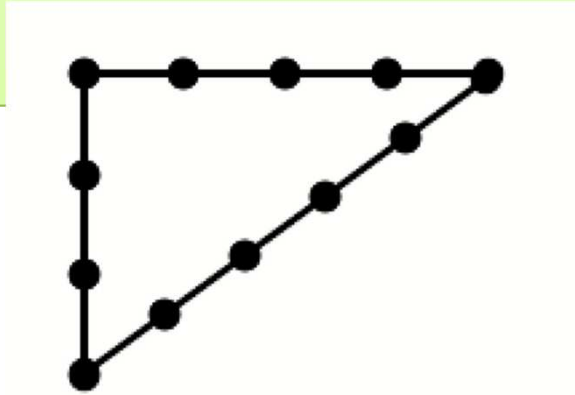


Diophantine optics

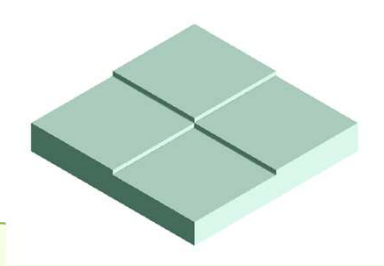
- *Diophantus of Alexandria*: greek mathematician, known as the « father of algebra »
- He studied polynomial equations with integer coefficients and integer solutions such as $(x-1)(x-2) = x^2 - 3x + 2 = 0$ called *diophantine equation*

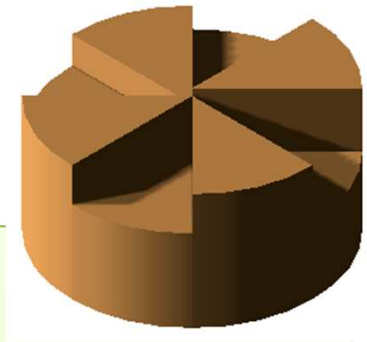
  The most famous one : the egyptian triangle $5^2 = 4^2 + 3^2$
- Optics and power of integers :
  - constructive or destructive interferences = optical path differences multiple integer (odd or even) of ⍰/2
  - complex amplitude = highly non-linear function of the opd

    ➜ Taylor development implies powers of integers
- Diophantine optics = exploitation in optics of some remarkable algebraic relations between powers of integers

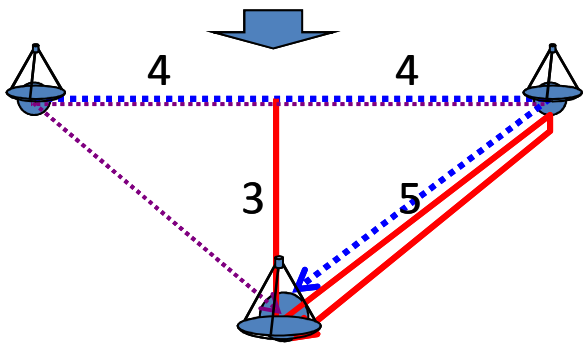


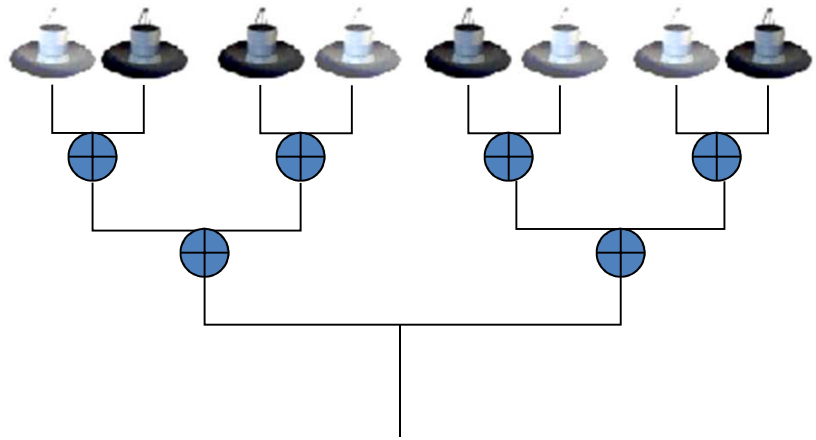
A few examples





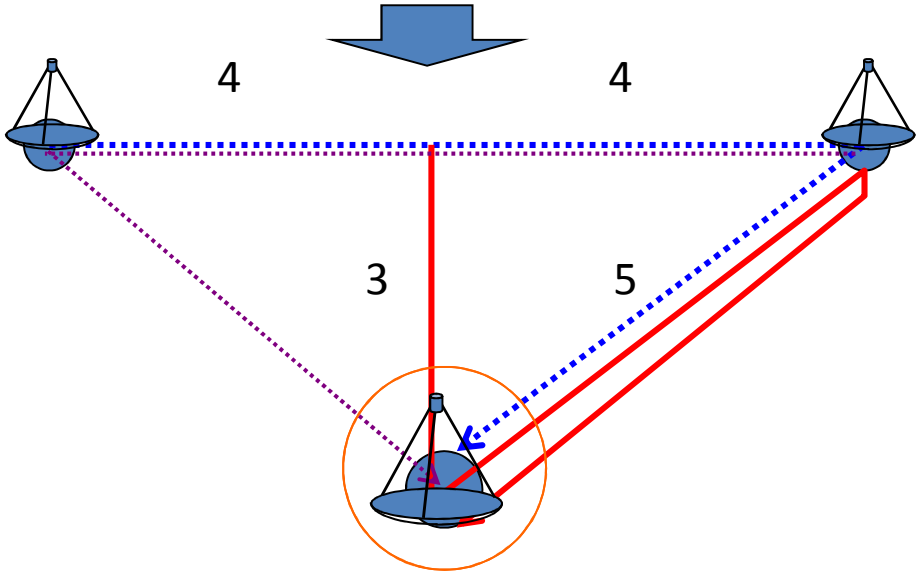
- Avoiding long delay lines in space interferometers using fixed mirrors
- Obtaining deep and flat nulling function θ^n in multi-telescopes nulling intererfometers
- Improving achromatism in phase mask coronagraph



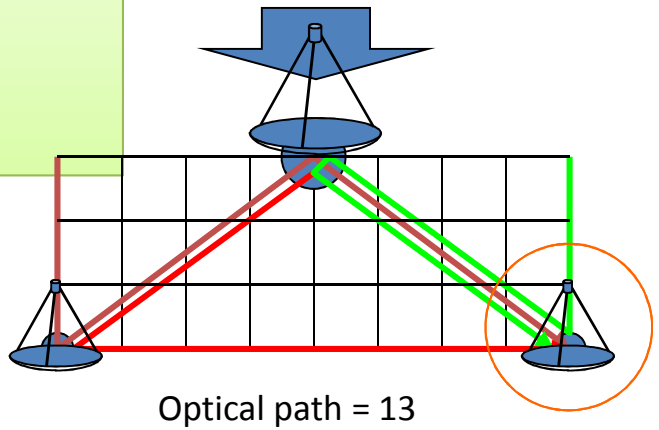


Avoiding long delay lines

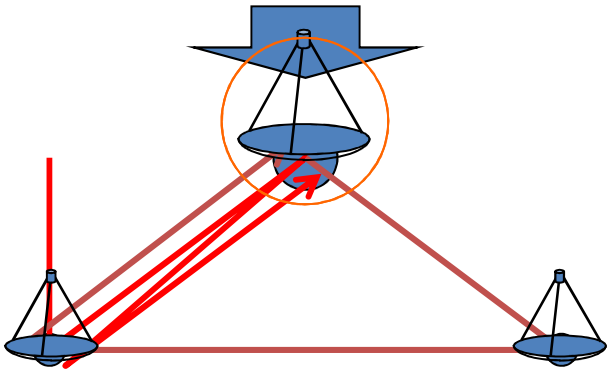
- Consider a 3-telescopes nulling interferometer in space
- How recombining the beams on one of the spacecraft carrying a telescope, without long delay lines which would be prohibitive in space ?
- Use relay mirrors and diophantine relations between spacecraft distances



Optical path = 4+4+5 = 3+5+5 = 13



Optical path = 13



Optical path = 18

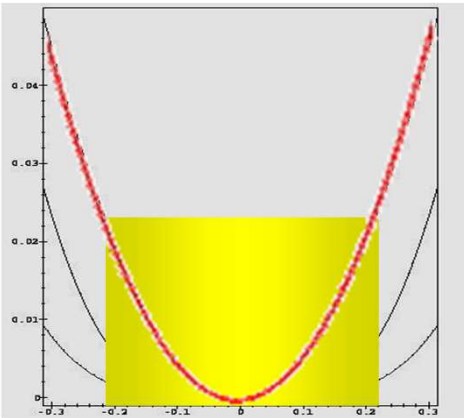
Deep nulling

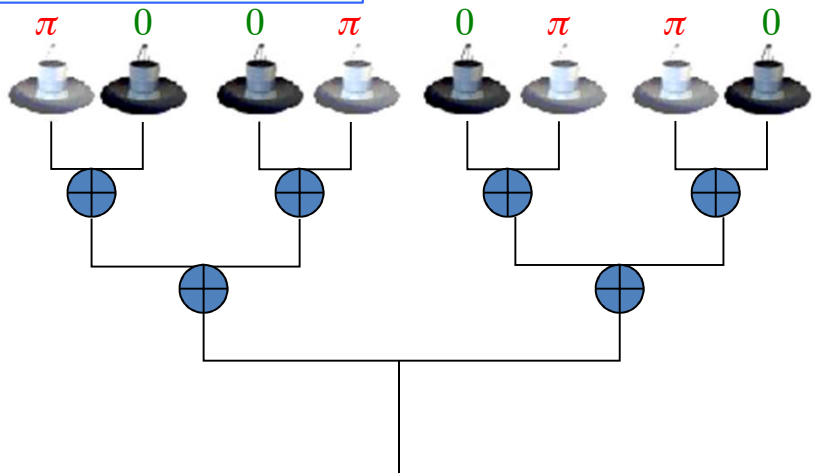
- In a Bracewell nulling interferometer, the stellar disk is resolved ➜ *leaks* of light
- To obtain a deep and flat nulling function in a multi-telescopes interferometer
  - distribute the telescopes with a π phase shift according to the Prouhet-Thué-Morse series 0110100110010110…
  - nulling varies then as θ^n with n as high as wished
- The coefficients of the Taylor development of the complex amplitude vanish thanks to diophantine relations between sums of powers of 2^L first integers

  0 1 1 0 1 0 0 1 …

  1 2 3 4 5 6 7 8 … 2^L $\quad \sum n_k^p = \sum m_k^p \quad$ for all $p < L-1$

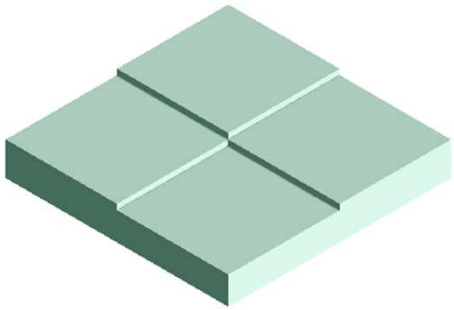
$$a = j\varphi \sum_k (m_k - n_k) - \varphi^2/2 \sum_k (m_k^2 - n_k^2) - j\varphi^3/6 \sum_k (m_k^3 - n_k^3) + \ldots$$

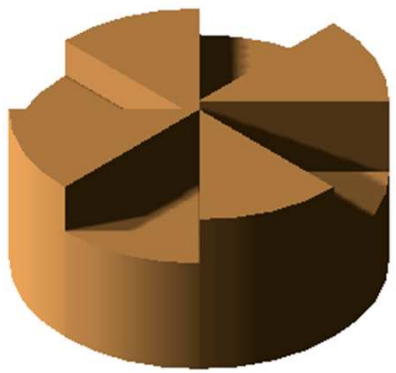




Achromatism of phase mask coronagraph

- Improving achromatism in phase mask coronagraph
- Play with combinations of quadrant thickness to cancel the first terms of the Taylor development of the complex amplitude ± exp(j k) where = k
  - 4QPMC : 2nd order with steps = [0, 1, 2, 1] a ≈ $(\quad)^2$
  - 8QPMC : 3rd order with steps = [1, 8, 3, 6, 2, 7, 2, 7] a ≈ $(\quad)^3$
- A 4QPMC component just manufactured : to be tested soon



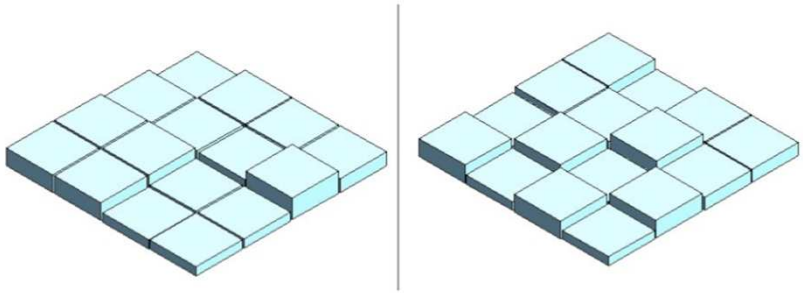


Achromatic phase shift in a nulling interferometer

The (diophantine) chessboard phase shifter

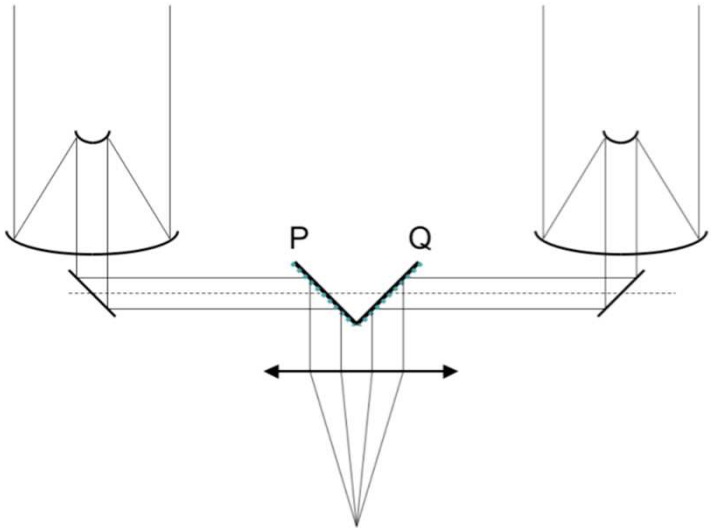
The chessboard phase shifter

- A new concept of achromatic phase shifter : the achromatic chessboard
  (Rouan & Pelat, A&A 2008 ; Pelat, Rouan, Pickel, A&A 2010 ; Pickel, Pelat, Rouan et al. , A&A 2013)
- Based on a single optical device and some unforeseen application of diophantine equations
- The wavefront is divided into many sub-pupils with two “chessboards” of phase-shifting cells, each producing an opd : an even or odd multiple of ?/2
- The proper distribution of opd produces the quasi-achromatization
- Main potential interest : makes the arms of the interferometer fully symmetric



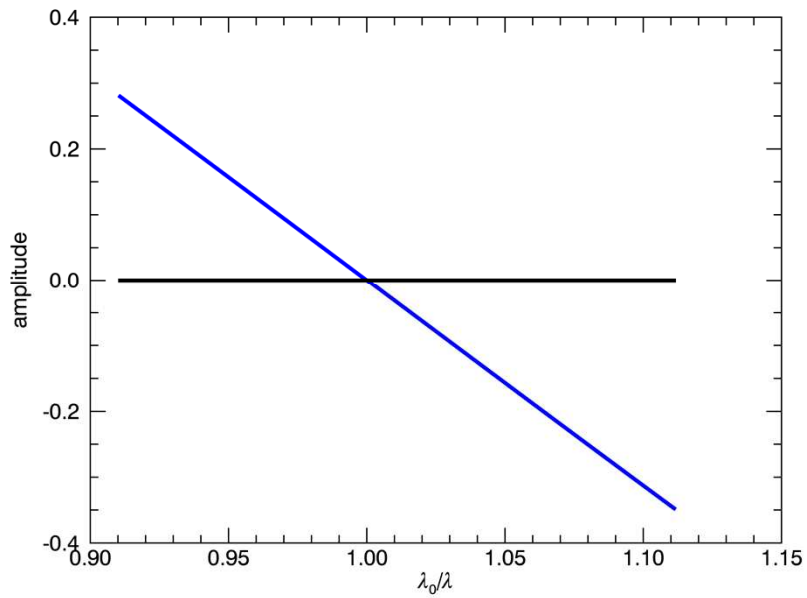
P (even pupil) Q (odd pupil)

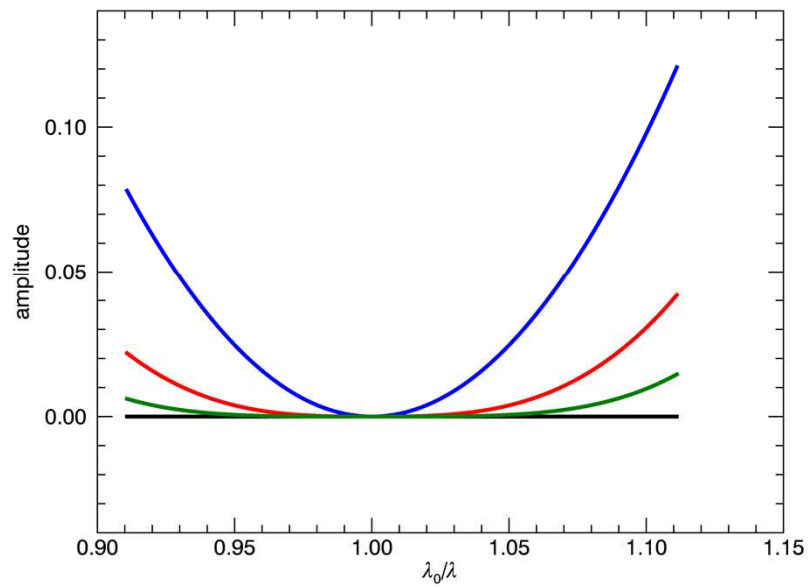
opd $= 2k \times \lambda_0/2$ opd $= (2k+1) \times \lambda_0/2$



opd distribution : some mathematics…

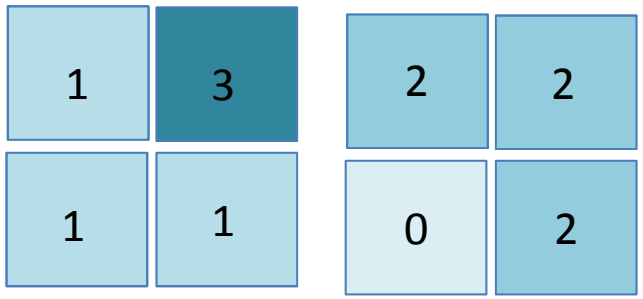
- If $z = (-1)^{\lambda 0/\lambda} \equiv e^{i\pi\lambda 0/\lambda}$
- the cell with *opd* = k $\lambda_0/2$ has a complex amplitude z^k
- For a basic Bracewell (one cell per chessboard) the amplitude is $\Lambda = 1 + z$
  That is $\lambda = \lambda_0$ induces a root of order one on Λ.
- To obtain a flat Λ around λ_0 let's produce a multiple root : $\Lambda = (1 + z)^n$
  - The higher *n* the deeper the nulling vs λ around λ_0

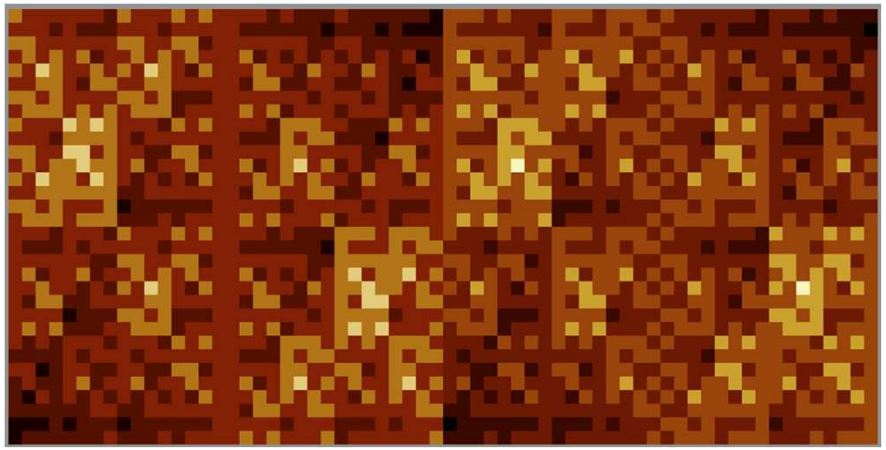




opd distribution : some mathematics…

- This generates the number of cells of a given phase shift according to the binomial coefficients
  - E.g. for n = 3, we get $(1 + z)^3 = 1 + 3z + 3z^2 + z^3 = 1 + z + z + z + z^2 + z^2 + z^2 + z^3$
  - producing 1 opd of 0, 3 of $\lambda 0 /2$, 3 of $2\lambda 0 /2$ and 1 of $3\lambda 0 /2$
  - And for a $2 \times 32 \times 32$ chessboard : $(1 + z)^{11}$

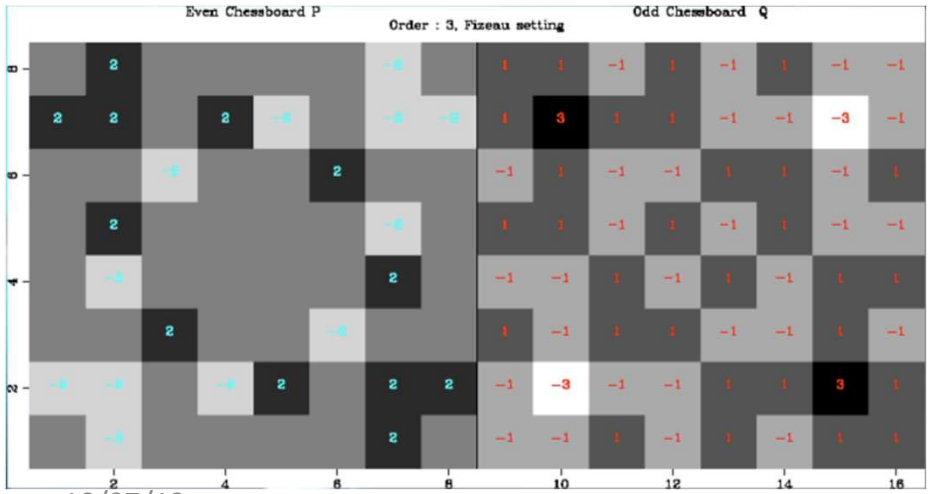




x,y-distribution of the steps

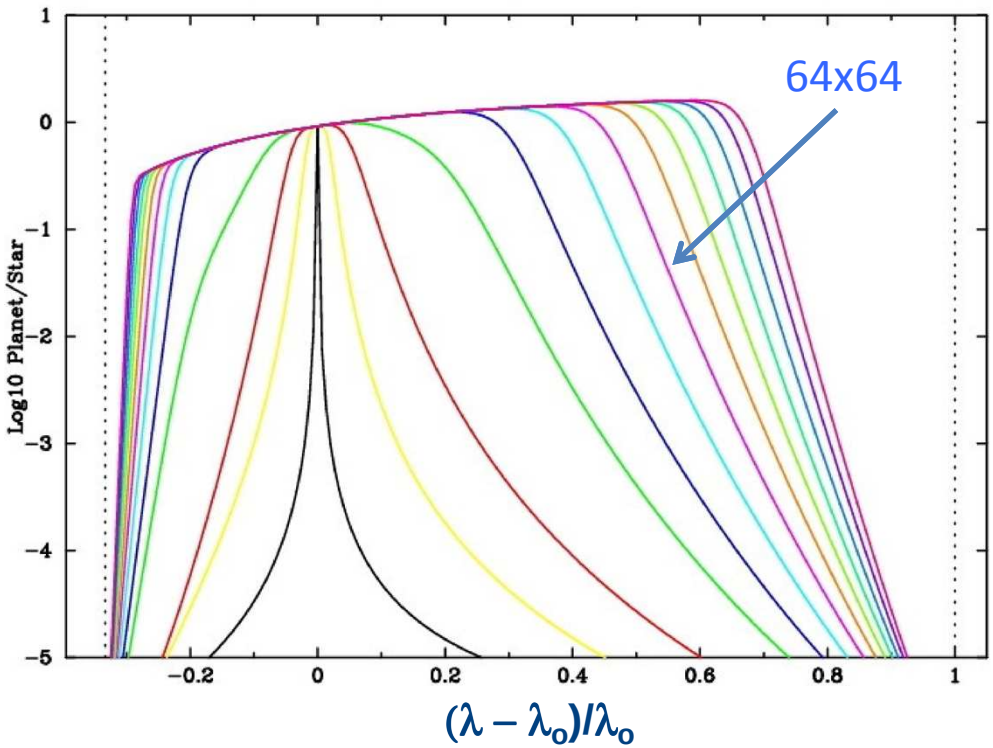
- P_r and Q_r = physical arrangement of the phase shifters at order
- The cells are placed in such a way that $P_r - Q_r$ is a finite difference differential operator of high order
- Light in the focal plane is rejected outside : this improves the nulling
- One can achieve this objective (Pelat, Rouan, Pickel, 2010) with the following iterative arrangement :

$$P_{r+1} = \begin{bmatrix} Q_r+1 & P_n+2 \\ P_r & Q_r+1 \end{bmatrix}, \quad Q_{r+1} = \begin{bmatrix} P_r+1 & Q_n+2 \\ Q_r & P_r+1 \end{bmatrix}$$



Performances of achromatization

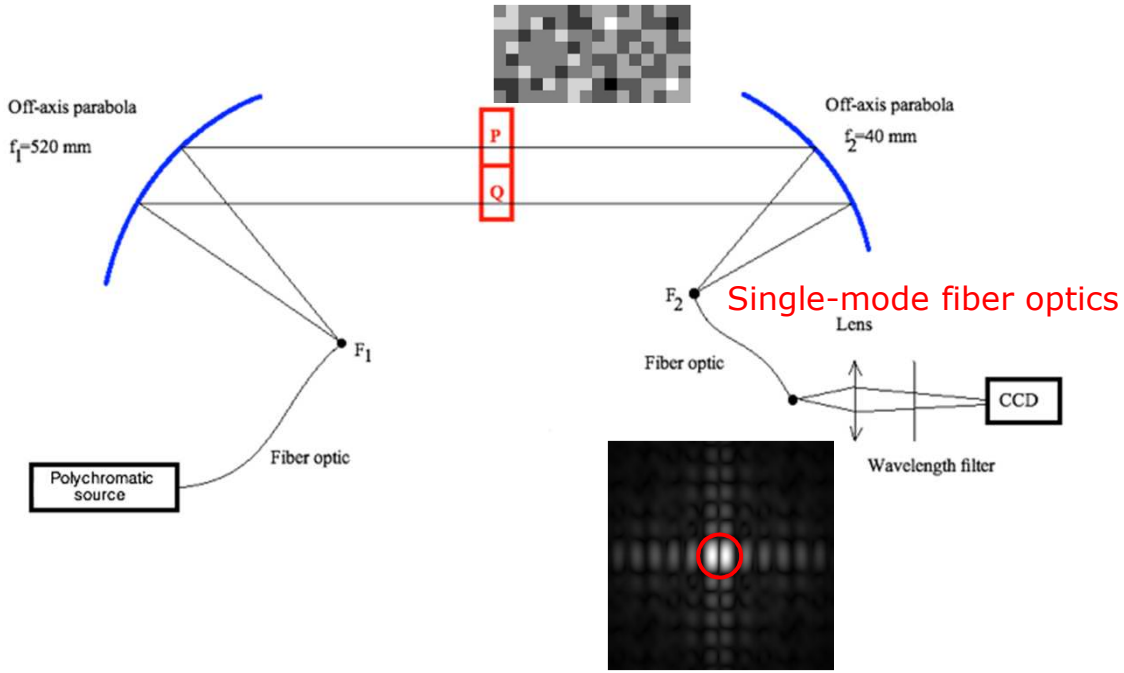
- Theoretical estimate :
- Absolute max bandpass : □□□□ = 2/3 □o — 2□o ➜ a factor 3 in □
- if 64x64 cells : □□□□ = (.65□o — 1.3□o) ➜ one octave.
  - Darwin specs (6 – 18 µm) achievable with two components



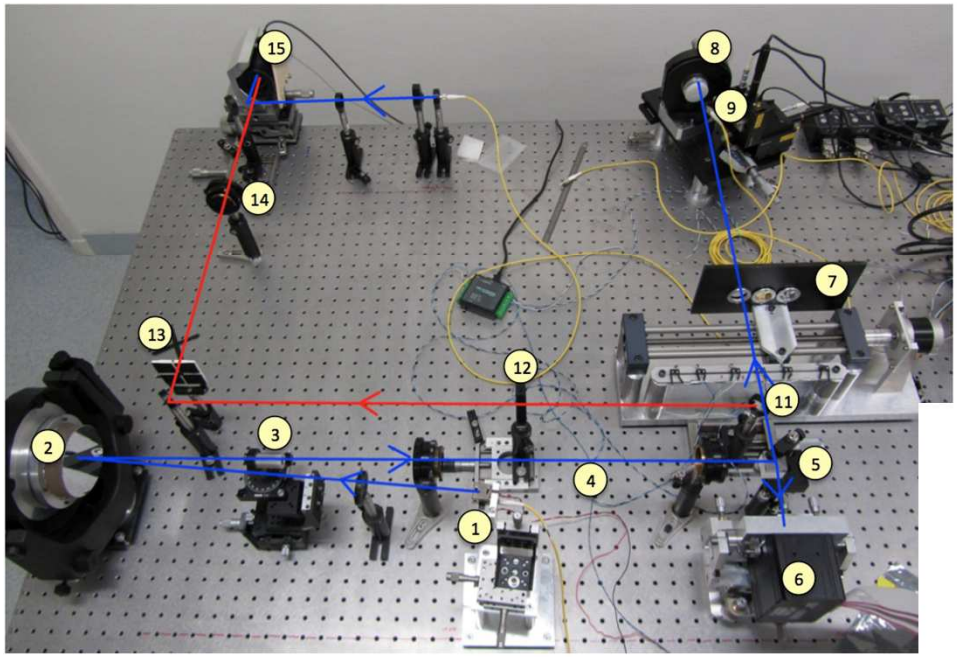
The DAMNED test bench*

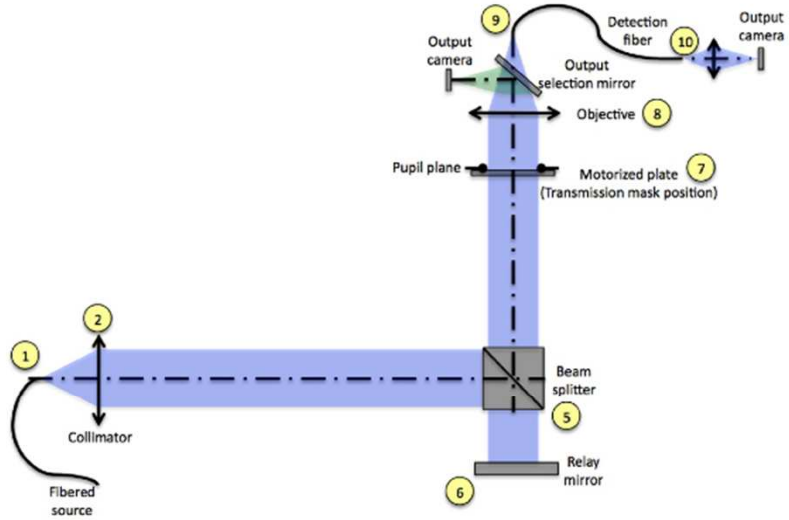
- Uses 2 off-axis parabola, a chessboard mask, a single-mode fiber
- Simulates two contiguous telescopes recombined in a Fizeau scheme
- The single-mode fiber is essential in the Fizeau scheme : it allows to sum the anti-symmetric amplitude and thus to make the nulling effective
- Measurement : x-y scanning of the focus with the fiber optics head

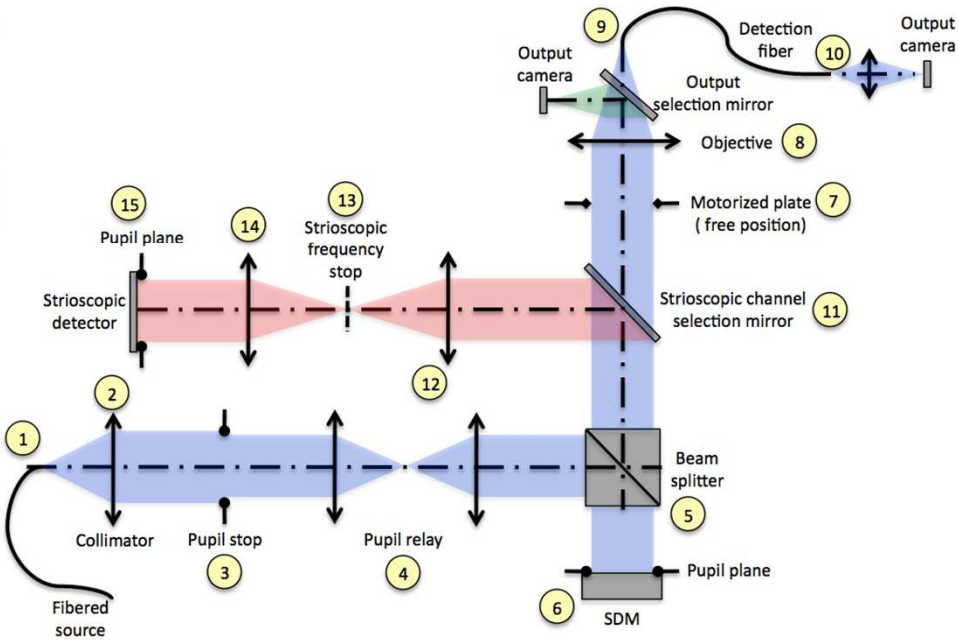
* Dual Achromatic Mask Nulling Experimental Demonstrator



The DAMNED test bench*

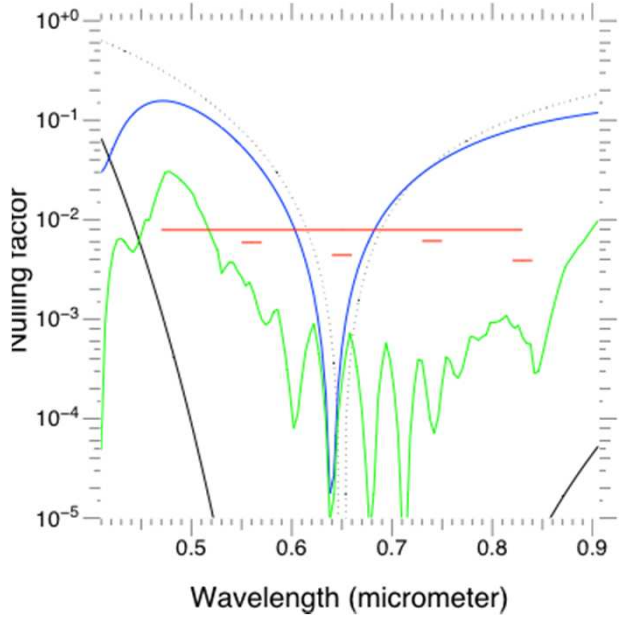


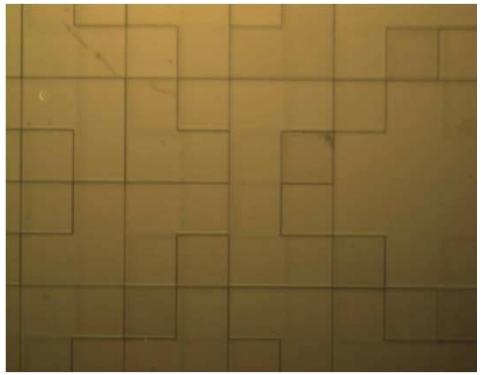


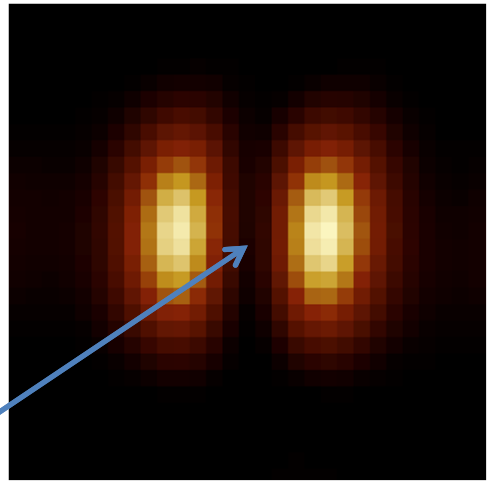


Transmissive chessboard mask

- Manufactured by GEPI – Obs de Paris
- 2 × 8 × 8 cells of 600 µm size in amorphous silica, using Reactive Ion Etching
- Result : typical nulling factor : 3-7 10^{-3} for broad-band filters ;
- Quasi-achromatism is indeed obtained : 8 10^{-3} in the range 460-840 nm
- The medium nulling factor does agree with simulations using the actual mask cell's thickness *: performances are limited by the mask's step accuracy*
- Need for a better accuracy of steps thickness





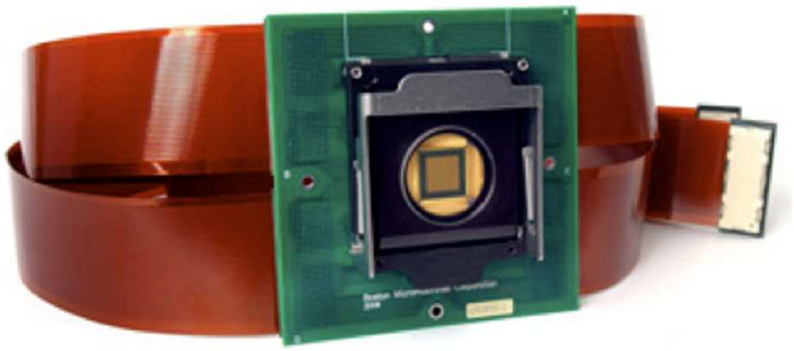


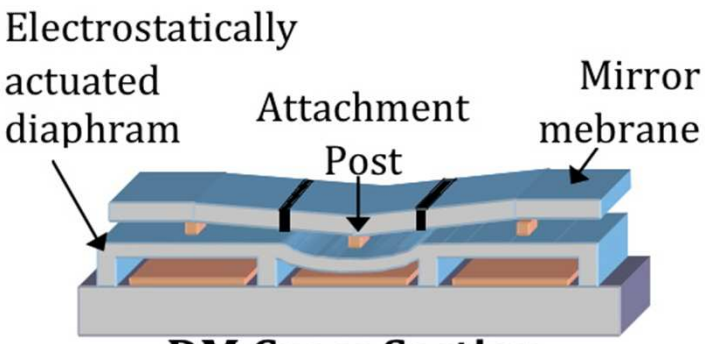
Nulling valley

Scan of the focus by the fiber head

Use of a segmented AO mirror

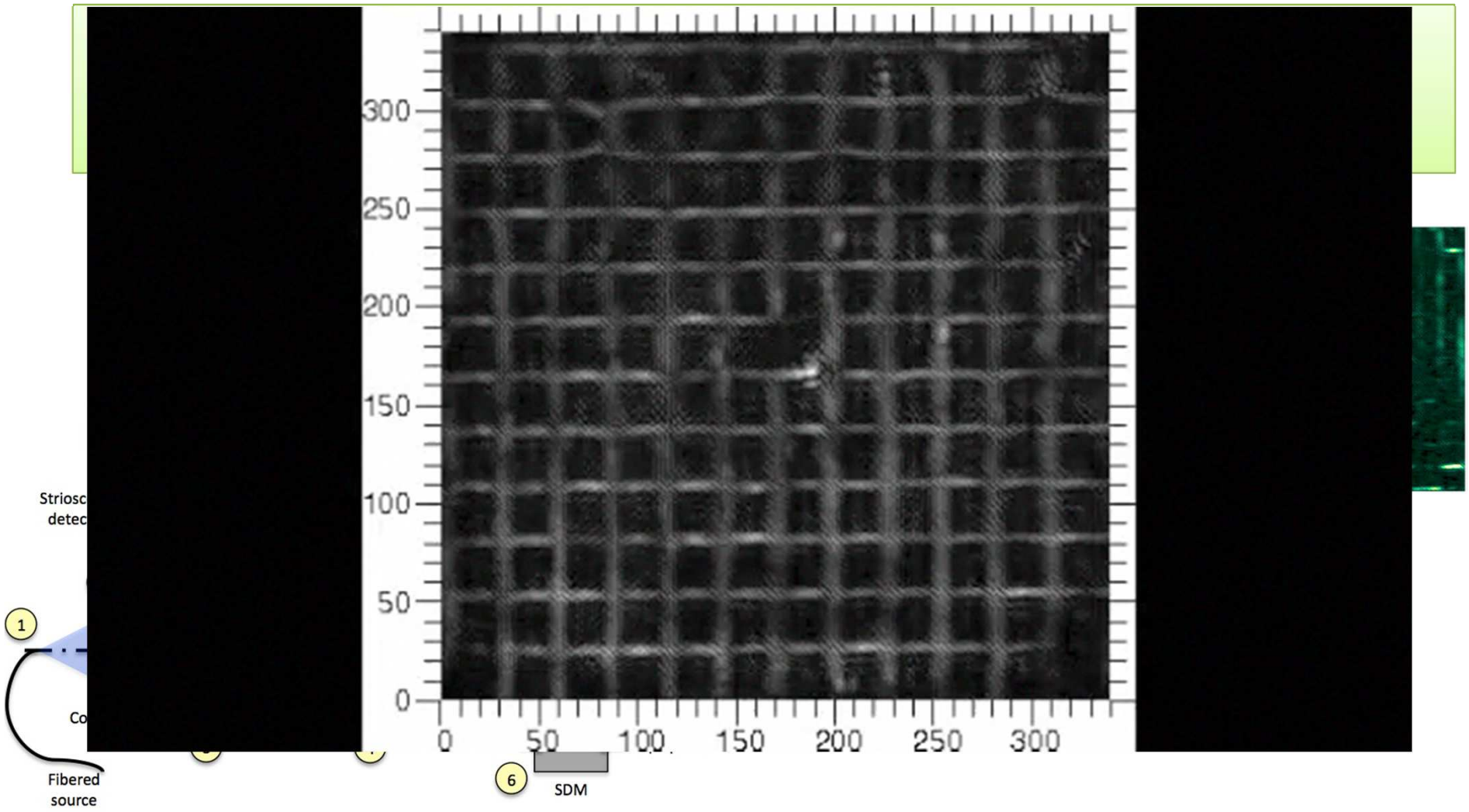
- Phase chessboard synthesized using a segmented *deformable mirror*:
  - free choice of the central wavelength
  - fine control of each cell's opd
  - versatile way to change the XY distribution
  - open the door to *modulation*
- Choice of a segmented Boston µ-machine 12 x 12 electrostatic mirror
- Optical scheme adapted to work in reflection
- Control of flatness using phase contrast





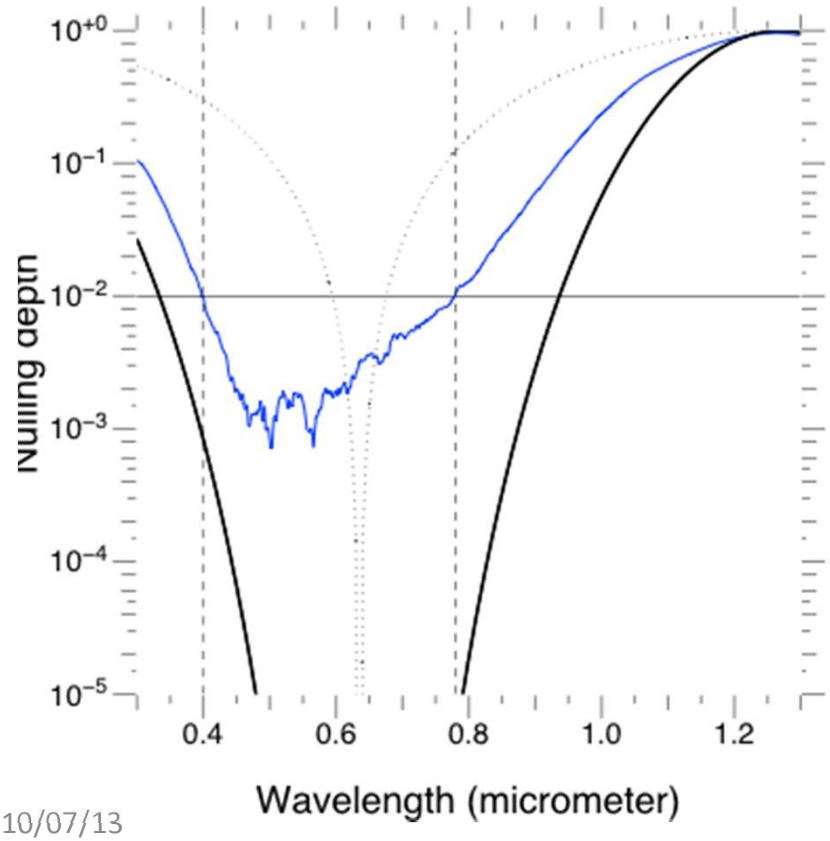
DM Cross Section

DM control using strioscopy



Performances w segmented AO mirror

- The parabola was no longer suited for the larger PSF (small DM) ➜ direct image of the *coffee bean* ➜ performance assessment less accurate
- Another method : scanning by the DM at a fixed λ of the source (laser)
- Performances similar to the transmissive chessboard while order is lower ($2 \times 4 \times 4$)



Work in progress

- Present work :
  - improvement of the piston accuracy
  - better optical design
  - spectroscopy for simultaneous chromatic performances

- Future work : modulation between different nulling configurations to measure possible biases

- Extrapolation to mid-IR :
  If the accuracy on piston is the same at 10 µm, a null depth of $2\ 10^{-6}$ would be reached on a low order ($2 \times 8 \times 8$) chessboard : within the specifications !

Conclusion - summary

- *Diophantine optics* : another way of thinking problems in optics
- In some cases it may bring a genuine solution
- Principle of the *achromatic chessboard phase shifter* demonstrated in the lab in reflection and transmission
- Performances and mode of operation using a *segmented deformable mirror* demonstrated : brings a clear asset

- *Diophantine optics also brings the best solution for sharing a salami between several fellow guests….*

abc bca cab bca cab abc cab abc bca bca cab

Thanks for your attention